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Abstract

Due to space-charge effect, there is a growth of bunch emittance across transition as a result of negative-mass instability. The models of growth at cutoff frequency and growth from high-frequency Schottky noise are reviewed. The difficulties of performing reliable simulations are discussed. An intuitive self-bunching model for estimating emittance growth is presented.

I. INTRODUCTION

As seen by the bunch particles, the space-charge force effectively switches sign while crossing transition. This leads to the mismatch of bunch length across transition and negative-mass instability after transition. Bunch-length mismatch results in the tumbling of the bunch in the longitudinal phase space and eventual emittance growth due to filamentation. Negative-mass instability is a type of microwave instability, which will also result in emittance growth and even possible breakup of the bunch. Although the effect of bunch-length mismatch can be cured by a quadrupole damper, [1] which eliminates the tumbling, the effect of negative-mass instability cannot be avoided except by a γ_t jump. In the Fermilab Main Ring or the future Main Injector, if the bunch 95% emittance grows to larger than 0.15 eV-sec, the efficiency of bunch coalescence will be affected with significant loss of particles. This article reviews the various treatments of emittance growth during negative-mass instability, and presents a simple intuitive self-bunching model to obtain an estimate.

II. GROWTH AT CUTOFF

In the absence of space charge or other coupling impedances, the motion of a particle in the longitudinal phase space can be derived analytically [2] at any time near transition in terms of Bessel function $J_{\frac{2}{3}}$ and Neumann function $N_{\frac{2}{3}}$. With the introduction of space charge, the growth rate of a small excitation amplitude can be evaluated analytically by integrating the Vlasov equation when the bunch has either an elliptical or bi-Gaussian distribution in the longitudinal phase space. The total growth can then be tallied up by small time steps across transition. Lee and Wang [3] made such a calculation for the Relativistic Heavy Ion Collider to be built at Brookhaven. The emittance growth was taken as two times the growth of the excitation amplitude at the

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cutoff frequency, and the result was considered satisfactory. The choice of the cutoff frequency comes from the assumption that electromagnetic waves emitted by the bunch at higher frequencies will not bounce back from the beam pipe to interact with the bunch. Wei [4] later studied the emittance growth of the Alternating-Gradient Synchrotron at Brookhaven using similar approach. His simulation showed that the emittance blowup had been very much overestimated. Wei pointed out that the bunch emittance had been kept constant by Lee and Wang in the computation of the growth for each time step. The bunch emittance was in fact growing and would provide more Landau damping to counteract the instability. With the emittance update at each time step, he found the numerical calculations agree with the simulations.

In order to discuss the shortcomings of the Lee-Wang-Wei method, let us first review some theory of the negative-mass instability. If we ignore Landau damping, the growth rate at peak current I_p at the revolution harmonic k_c is given by

$$G(k_c,t) = \omega_0 \left(\frac{\eta e k_c Z_I I_p}{2\pi\beta^2\gamma E_0} \right)^{1/2} , \tag{2.1}$$

where $\omega_0/2\pi$ is the revolution frequency, E_0 the particle rest energy, η the frequency-slip parameter, e the particle charge, t the time measured from the moment of transition crossing, and Z_I the imaginary part of the space-charge impedance, which is given by

$$\frac{Z_I}{k_c} = \frac{Z_0 g}{2\beta\gamma^2} . \tag{2.2}$$

Here, $Z_0 \approx 377$ ohms is the free-space impedance, γ and β the relativistic parameters of the bunch particle at or near transition, and g the space-charge geometric parameter, which at zero frequency can be expressed in terms of the beam pipe radius b and beam radius a as

$$g_0 = 1 + 2\ln\frac{b}{a} , \tag{2.3}$$

and rolls off at high frequencies roughly like

$$g = \frac{g_0}{1 + (k_c/k_{c\frac{1}{2}})^2} \tag{2.4}$$

when b/a is not too big, with the half-value harmonic given by

$$k_{c\frac{1}{2}} = \gamma R \left(\frac{1.6}{b} + \frac{0.52}{a} \right) \tag{2.5}$$

and R the radius of the accelerator ring. It is clear from Eq. (2.1) that at frequencies below the rolloff of the space-charge impedance, the growth rate for negative-mass instability is directly proportional to the harmonic k_c. When Landau damping is taken into account, Hardt [7] showed that the growth rate becomes

$$G(k_c,t) \propto k_c g^2 , \tag{2.6}$$

which provides a maximum at $k_{cp} = k_{c\frac{1}{2}}/3^{\frac{1}{2}}$. Taking the Fermilab Main Ring as an example, this corresponds to 77.6 GHz when $a = 5$ mm and $b = 35$ mm. On the other hand, the cutoff frequency is only about 1.5 GHz. For a typical cycle, the total growth across transition for a power spectral line is 1.5×10^6 times at the former frequency but only 1.6 at the latter frequency. As a result, it is difficult to justify the correctness of the description of Lee-Wang-Wei. In addition, in Wei's simulation, the bunch was divided into bins with the bin width equal to the cutoff wavelength. In other words, all large-growth-rate amplitudes at high frequencies had been neglected. Here, we want to point out that the first simulation across transition to show negative-mass instability was done by Lee and Teng [5] on the Fermilab Booster, where they also divided the bunch up into cutoff wavelengths only. Later, simular simulations on the same Booster were performed by Lucas and MacLachlan, [6] and they also failed to include the high-frequency amplitudes.

One may raise the question that a typical proton bunch which is usually much longer than the radius of the beam pipe will have a spectrum not much higher than the cutoff frequency. In order to have a growth at harmonic $k_c = k_{cp}$ or $k_{c\frac{1}{2}}$, the original amplitude or the seed of the growth has to be supplied by Schottky noise, which is extremely small, so that the growth effect to the bunch at such high frequencies may not be significant. This question will be discussed in Sec. IV below, after we go over the Schottky-noise model of Hardt. [7]

III. SCHOTTKY-NOISE MODEL

Hardt assumed that the seeds of the negative-mass growth are given by the statistical fluctuations of the finite number of particles N_b within the bunch. The bunch is divided into M bins in the rf phase coordinate ϕ. According to the smooth distribution $F(\phi)$ which is normalized to have an average of unity, there are $N_bF(\phi)\Delta\phi/2\hat{\phi}$ particles, where $\Delta\phi = 2\hat{\phi}/M$ is the bin width and $\hat{\phi}$ is the half length of the bunch. Due to statistical fluctuation, the mth bin contains δN_m extra particles. So a step function $f(\phi)$ can be defined:

$$f(\phi,t) = \frac{\delta N_m}{\Delta N} \qquad \text{if} \quad \frac{m-1}{M} < \frac{\phi+\hat{\phi}}{2\hat{\phi}} < \frac{m}{M} \tag{3.1}$$

where $\Delta N = N_b/M$ is the average number of particles in a bin. Expanding in a Fourier series

$$f(\phi,t) = \sum_{k_b=-\infty}^{\infty} c_{k_b}(t)e^{i2\pi k_b\phi/(2\hat{\phi})}\,, \tag{3.2}$$

it can easily be shown that the initial expectation is

$$E(|c_{k_b}(0)|^2) = \frac{1}{N_b}\,, \tag{3.3}$$

independent of mode number k_b and the number of bins M. The growth rate of each mode amplitude c_{k_b} can be derived from the Vlasov equation, and the evolution is

$$|c_{k_b}(t)| \approx \frac{1}{\sqrt{N_b}} \exp \int_0^t G(k_c, t) dt \,, \tag{3.4}$$

where the growth rate has been given in Eq. (2.6). Here, the bunch mode number k_b should not be confused with the revolution harmonic number k_c. They are related to each other by

$$\frac{k_b}{k_c} = \frac{2\hat{\phi}}{2\pi h} \,, \tag{3.5}$$

where h is the rf harmonic. Note that the growth rate $G(k_c, t)$ is odd in k_c or k_b, so that half the bunch modes grow and half of them are damped. In other words, we need to take care of modes with one sign of k_b only.

Hardt made the assertion that there is no blowup if

$$\sum_k |c_k(t_0)|^2 < 1 \,, \tag{3.6}$$

where t_0 is the time when stability is regained. From this, a threshold for blowup can be formulated. For example, a critical parameter c can be defined by [8]

$$\xi k_{cp} \left(\frac{r_p}{R}\right)^2 \left(\frac{E_0^{5/2}}{h^{1/3} \omega_0^{4/3} \gamma_t^{2/3}}\right) \left(\frac{N_b^2 g_0^2 |\tan \phi_s|^{1/3}}{A^{5/2} \dot{\gamma}_t^{7/6}}\right) = c E_c \,, \tag{3.7}$$

where E_c is the maximum allowable time integrated growth, and is given by

$$E_c = \frac{1}{2} \left[\ln N_b - \ln \left(\frac{k_{b\frac{1}{2}} \sqrt{8\pi}}{3\sqrt{\frac{1}{2} \ln N_b}} \right) \right] \,. \tag{3.8}$$

When the critical parameter $c < 1$, there is no blowup. In above, the coefficient $\xi = 2^{-41/6} 3^{25/6} \pi^2 \Gamma(2/3)(1 - \pi/4)$, A is the area or emittance of an elliptical bunch in eV-sec, r_p is the classical proton radius, ϕ_s is the synchronized rf phase, γ_t is γ at transition, $\dot{\gamma}_t$ is the rate at which transition is crossed, and $k_{b\frac{1}{2}}$ is the half-value bunch mode number, which is related to the half-value revolution harmonic by $k_{b\frac{1}{2}} = k_{c\frac{1}{2}} \hat{\phi}/\pi h$.

Some comments are in order:

(1) The critical condition $\sum |c_k(t_0)|^2 = 1$ implies that

$$\frac{1}{2\hat{\phi}} \int |f(\phi, t_0)|^2 d\phi = 1 \tag{3.9}$$

or the average fluctuation in each bin is comparable to the average number of particles in each bin, which is really a large particle fluctuation or a big blowup in the bunch. This blowup implies violent changes in the bunch, such as total bunch breakup. However, the assertion of Eq. (3.5) is a bit hand-waving, because even when the average fluctuation is much less than unity, there can be a big blowup of the bunch emittance already. Hardt's paper provides no recipe to compute the increase in bunch emittance in this regime.

(2) The perturbation expansion is in fact

$$F(\phi)+f(\phi,t)=F(\phi)+\sum_{k_b=-\infty}^{\infty}c_{k_b}(t)e^{i2\pi k_b\phi/(2\hat{\phi})}\,, \tag{3.10}$$

where the unperturbed distribution has an average of unity. Since Hardt only studied the situation of no blowup or when the fluctuation function $f(\phi,t)$ has an average of less than unity, the perturbation is justified although the amount of growth of the c_{k_b}'s is tremendous.

IV. COMPARISON OF GROWTHS AT CUTOFF AND HIGH FREQUENCIES

For a parabolic bunch, the unperturbed linear distribution is

$$F(\phi)=\frac{3}{2}\left(1-\frac{\phi^2}{\hat{\phi}^2}\right)\,. \tag{4.1}$$

When it is expanded in a Fourier series

$$F(\phi)=\sum_{k_b=-\infty}^{\infty}a_{k_b}(0)e^{i2\pi k_b\phi/(2\hat{\phi})}\,, \tag{4.2}$$

the mode amplitude is

$$a_{k_b}(0)=\frac{3}{\pi^2}\frac{(-1)^{k_b+1}}{k_b^2}\,. \tag{4.3}$$

The bunch mode number k_b which corresponds to the cutoff harmonic $k_c=R/b$ can be estimated using Eq. (3.5). Then, the final value of a power spectral line can be computed:

$$|a_{k_b}(t_0)|^2=|a_{k_b}(0)|\exp\int_0^{t_0}2G(k_c,t)dt\,. \tag{4.4}$$

The results are listed in Table I for various run cycles of the Fermilab Main Ring. The beam pipe radius and the beam radius are kept fixed at $b=35$ mm and $a=5$ mm, respectively. The synchronous phase is 60°. Alongside, we have also tabulated the final size of the Schottky power spectral line at the high harmonic k_{cp} according to Eq. (3.4). The sum of all the Schottky power spectral modes was derived by Hardt to be

$$\sum_{k_b}|c_{k_b}(t_0)|^2\approx|c_{k_b}(t_0)|^2\Big|_{k_c=k_{cp}}\times\frac{k_{b\frac{1}{2}}}{3}\left(\frac{8\pi}{E_p}\right)^{\frac{1}{2}}\,, \tag{4.5}$$

where

$$E_p=\int_0^{t_0}G(k_{cp},t)dt \tag{4.6}$$

$\dot{\gamma}_t$ sec^{-1}	N_b 10^{10}	Initial Bunch Emittance eV-sec	Final Power Spectrum of Fluctuation at $k_{\rm cutoff}$	at k_{cp}	sum
90	2.2	0.05	3.70	1.50×10^{9}	4.03×10^{10}
90	2.2	0.06	2.21	1.08×10^{2}	3.97×10^{3}
90	2.2	0.07	1.67	1.19×10^{-2}	5.74×10^{-1}
90	2.2	0.08	1.41	4.86×10^{-5}	2.93×10^{-3}
90	2.2	0.09	1.26	1.41×10^{-6}	1.06×10^{-4}
120	4.0	0.06	7.44	4.37×10^{18}	1.00×10^{20}
120	4.0	0.07	3.80	1.94×10^{9}	5.83×10^{10}
120	4.0	0.08	2.54	4.40×10^{3}	1.67×10^{5}
120	4.0	0.09	1.95	1.02×10^{0}	4.76×10^{1}
120	4.0	0.10	1.64	3.57×10^{-3}	2.00×10^{-1}

Table I: Final fluctuation power spectra at cutoff and high-frequency Schottky harmonics

is the integrated growth at the peak harmonic k_{cp}. This is also listed in the last column of the table.

We can see that the Hardt's blowup criterion of Eq. (3.6) appears to be critical, where the growth changes tremendously. When that criterion is exceeded, the Schottky modes are always larger than the mode at cutoff, showing that the inclusion up to cutoff frequency is inadequate. On the other hand, below the blowup limit, the mode at cutoff is larger than the high-frequency Schottky modes, implying that there should be modest emittance growth below the Hardt's blowup limit. However, this does not tell us how large the emittance growth is. It will be best if we can sum up the final power spectrum of the bunch distribution:

$$\sum_k |a_{k_b}(t_0)|^2 = \sum_{k_b} \frac{9}{\pi^4 k_b^4} e^{\text{integrated growth}} . \tag{4.7}$$

Unfortunately, this sum is divergent because the integrated growth is directly proportional to k_b. Even when we take into account the space-charge rolloff, the sum still becomes unreasonably large. The reason behind this is the breakdown of the linear perturbation when the perturbed spectral mode becomes larger than the unperturbed one. As a result, it remains unclear whether the high-harmonic Schottky noise is dominating in the growth of the bunch emittance. A simulation seems to be the best solution.

V. DIFFICULTIES IN SIMULATION

A simulation of the negative-mass instability is not trivial. There are two main difficulties:

(1) Inclusion of high-frequency components

The growth of the Schottky noise peaks at k_{cp}, which corresponds to roughly 78 GHz for the Fermilab Main Ring, while the half-value space-charge rolloff harmonic $k_{c\frac{1}{2}}$

corresponds to 134 GHz. Therefore, in simulations we need a bin size of about 1/134 or 0.0075 ns. The tracking code ESME [9] developed at Fermilab divides the whole rf wavelength or 18.8 ns up into 2^n bins where n is an integer, and the number of bins will have to be at least 4096 which is too large. As a rule of thumb, the bins should have a width less than a/γ, where a is the beam radius. Simulations of the Main Ring across transition had been performed using ESME. As we increase the bin number from 128 to 256 and 512, we do see self-bunching in the phase plot corresponding to the highest frequency of 3.40, 6.81, and 13.6 GHz, respectively, in each of the situations, as illustrated in Fig. 1. This suggests that the negative-mass growths at the high Schottky frequencies do play a role across transition. [10] In an actual simulation, the space-charge force is usually implemented by a differentiation of the bunch profile. To maintain the same numerical accuracy, we need to follow the "three-in-one rule," [11] which states that whenever the bin width is reduced by a factor of 2, the number of macro-particles needs to be increased by a factor of 2^3. As a result, the tracking time will increase by a factor of 2^4.

However, a typical Main Ring bunch has a full length of only 1 ns at transition. If we divide just two or three times the bunch region into bins, there will be only 256 or 512 bins, which will reduce the tracking time drastically. Søfrenssen [12] had successfully performed simulation with a bin width of a/γ. But he did not overcome the second difficulty that we are going to discuss next.

(2) The right amount of Schottky noise

In a simulation of microwave instability, there is usually ample time for the instability to develop to saturation. Therefore, we do not care so much about the size of the initial excitation or seed of the growth. Across transition, however, the bunch is negative-mass unstable only for a short time until the frequency-flip parameter η becomes large enough to provide enough Landau damping, and this time is typically of the order of the non-adiabatic time, which is about 3 ms for the Fermilab Main Ring. Therefore, the initial excitation amplitude needs to be tailored exactly. To have the exact Schottky noise level, we need to use in the simulation micro-particles instead of macro-particles. The Fermilab Main Ring bunch has typically $N_b = 2.2 \times 10^{10}$ particles, which is certainly unrealistically too many in a simulation.

A suggestion is to populate the bunch by N_M macro-particles according to a Hammersley sequence [13] instead of randomly. Then, the number of particles in each bin in excess of the smooth distribution will be $\mathcal{O}(1)$ initially, or the fluctuation function defined in Eq. (3.1) starts from $f(\phi, 0) = F(\phi)/\Delta N = MF(\phi)/N_b$. The expectation of the initial bunch mode amplitude turns out to be

$$E(|c_{k_b}(0)|^2) = \frac{M}{N_M^2}, \tag{5.1}$$

where M is the number of bins. Comparing with Eq. (3.3) for a randomly distributed bunch, the required number of macro-particles becomes

$$N_M = (MN_b)^{\frac{1}{2}}\,, \tag{5.2}$$

which is more reasonable (~ 2.4 to 3.6×10^6), but may be still too large to be managed in a simulation.

There are, however, two other difficulties with the Hammersley-sequence method. The step function $f(\phi, t)$ defined in Eq. (3.1) has an initial expectation of

$$E[f(\phi, 0)] = \frac{MF(\phi)}{\sqrt{N_b}}\,, \tag{5.3}$$

for the mth bin at ϕ_m, which is proportional to the initial unperturbed bunch distribution $F(\phi)$. Now it changes to $E[f(\phi, 0)] = \frac{M}{N_M}$ which is independent of $F(\phi)$. Thus, the relative fluctuations in the bins cannot be made to resemble those in the randomly populated bunch, and the initial fluctuation spectrum would have been altered.

In order to have the bunch to fit the space-charge modified rf bucket before transition, we usually switch on the space-charge force adiabatically over many synchrotron periods so that the initial populated bunch emittance will be preserved. For bunch particles distributed with a Hammersley sequence, the favored Hammersley statistics can often be lost after several synchrotron oscillations. A test was performed with 2×10^5 particles in a truncated bi-Gaussian distribution. The bunch was projected onto one coordinate, where it was divided into 20 equal bins. To simulate synchrotron oscillation, the bunch was then rotated in phase space with an angular velocity which decreases linearly by 1% from the center to the edge of the bunch. The fluctuation or number of particles in excess of the smooth projected Gaussian distribution in each bin was recorded for every rotation, and the rms was computed. The results are plotted in Fig. 2 as a function of rotation number. We see that although the rms fluctuation starts from 7 initially, it increases rapidly to ~ 12 after 5 rotations, ~ 20 after 20 rotations, and will approach its statistical value of 100 eventually. This might have been an overestimation, because the actual decrease in synchrotron frequency is not linear and the decrease near the core of the bunch where most particles reside is very much slower. Nevertheless, this test gives us an illustration of restoration to randomness. To cope with the fast restoration to randomness, one possibility is to compute exactly the initial distribution of the bunch in the space-charge modified rf bucket right at transition and populate the bunch according to a Hammersley sequence. In this way, the tracking of the bunch particles across the negative-mass unstable period, which is usually of the order of one synchrotron period, may reveal the reliable growth from the correct Schottky noise level.

VI. SELF-BUNCHING MODEL

Microwave instability can be viewed as self-bunching. The beam current I_p, seeing the impedance Z_I, gives rise to an rf voltage I_pZ_I, and creates a self-bunching rf bucket with an energy half height

$$\frac{\Delta E}{E} = \left(\frac{e I_p Z_I}{2\pi \eta k_{cp} \gamma E_0} \right)^{\frac{1}{2}} , \tag{6.1}$$

where k_{cp} denotes the revolution harmonic of the impedance. If this bucket height is less than the energy spread of the bunch, there will not be any extra energy spread and the bunch will be stable. If the bucket height is larger than the energy spread of the bunch, the bunch particles will travel outside the original energy boundary of the bunch, giving rise to an emittance growth as a result of filamentation. In fact, this is just another way of expressing the Keil-Schnell criterion. [14]

Here, we want to make the conjecture that this self-bunching bucket height determines the final energy spread of the bunch. Inside this bucket, the angular synchrotron frequency is given by

$$\omega_s = \left(\frac{k_{cp} \eta I_p Z_I}{2\pi \beta^2 \gamma E_0} \right)^{\frac{1}{2}} \omega_0 . \tag{6.2}$$

Since the frequency-flip parameter η is changing rapidly at transition, we substitute

$$\frac{\eta}{\gamma} = \frac{2\dot{\gamma}_t}{\gamma_t^4} t . \tag{6.3}$$

Integrating Eq. (6.2), we obtain the time to reach a quarter of a synchrotron oscillation from the moment of transition crossing as

$$T = \left(\frac{3\pi}{4} \right)^{\frac{2}{3}} \left(\frac{\pi E_0 \gamma_t^4}{k_{cp} I_p Z_I \dot{\gamma}_t \omega_0^2} \right)^{\frac{1}{3}} . \tag{6.4}$$

This will be the time required for some particles to reach the top of the bucket. Of course, the height of the self-bunching bucket is also changing, and the value of $\eta\gamma$ at this moment should be substituted in Eq. (6.1). At this moment, the unperturbed energy spread of an elliptical bunch with emittance A and without space-charge distortion is [15]

$$\frac{\Delta E}{E} = \frac{\Gamma(1/3)}{2^{1/2} 3^{1/6} \pi} \left(\frac{A \beta^2 \gamma_t^2}{E_0 T_c^2 \dot{\gamma}_t} \right)^{\frac{1}{2}} \left(1 - \frac{2\pi}{3^{1/6} \Gamma^2(1/3)} \frac{T}{T_c} \right) , \tag{6.5}$$

where

$$T_c = \left(\frac{\beta^2 \gamma_t^4 |\tan \phi_s|}{2h\omega_0 \dot{\gamma}_t^2} \right)^{\frac{1}{3}} . \tag{6.6}$$

is the non-adiabatic time. The correction in the second term of Eq. (6.5) is usually small. Thus, the growth in energy spread can be computed easily, and assuming filamentation the growth in emittance can be obtained. This estimate will be valid if T is less than the time to regain stability. The growths for some situations of the Fermilab Main Ring

$\dot{\gamma}_t$ sec^{-1}	N_b 10^{10}	Initial Bunch Emittance eV-sec	Fractional Emittance Growth	
			Self-Bunching Model	Cutoff Model
90	2.2	0.05	4.09	4.06
90	2.2	0.06	3.03	2.43
90	2.2	0.07	2.35	1.83
90	2.2	0.08	1.89	1.54
90	2.2	0.09	1.52	1.38
120	4.0	0.06	5.32	8.16
120	4.0	0.07	4.12	4.17
120	4.0	0.08	3.31	2.78
120	4.0	0.09	2.72	2.14
120	4.0	0.10	2.29	1.80

Table II: Growth of emittance for the self-bunching and growth-at-cutoff models.

are given in Table II. The corresponding growths obtained from the growth-at-cutoff model are also listed for comparison.

There is at present no reliable simulation of emittance growth. Experimental measurements are also marred with other mechanisms, such as bunch tumbling due to bunch-length mismatch, particles with different momentum crossing transition at different time, etc. Another example at the Fermilab Main Ring is that the bunch emittance usually grows to such a value that scraping occurs. Therefore, it is hard to judge at this moment the reliability of this crude model. On the other hand, this model can certainly be improved to a certain degree by including, for example, the space-charge distortion of the bunch shape, the tilt effect in phase space near transition, as well as the mechanism of overshoot when stability is regained.

VII. CONCLUSION

The negative-mass instability across transition was discussed. We reviewed the growth-at-cutoff model for emittance increase as well as Hardt's model of emittance blowup from high-frequency Schottky noise. We concluded that these two models fail to provide us with the correct emittance growth. The difficulties of performing a reliable simulation across transition were outlined. Finally, a self-bunching model for estimating emittance growth was presented. At this moment, no reliable simulation results and experimental results are available for comparison.

There remain some problems which require further investigation. The expression of the space-charge impedance given in Eq. (2.4) has been derived by assuming the fact that the phase velocity is equal to the particle velocity. This assumption may not hold at extremely high frequencies, and its breakdown may affect the behavior of the space-charge impedance at high frequencies. The Keil-Schnell criterion for a bunch beam which provides the threshold for microwave instability may not be valid near transition crossing. This is possible because synchrotron oscillation is extremely slow there. This may affect the basis of the crude self-bunching estimate. These problems will be examined in a separate paper.

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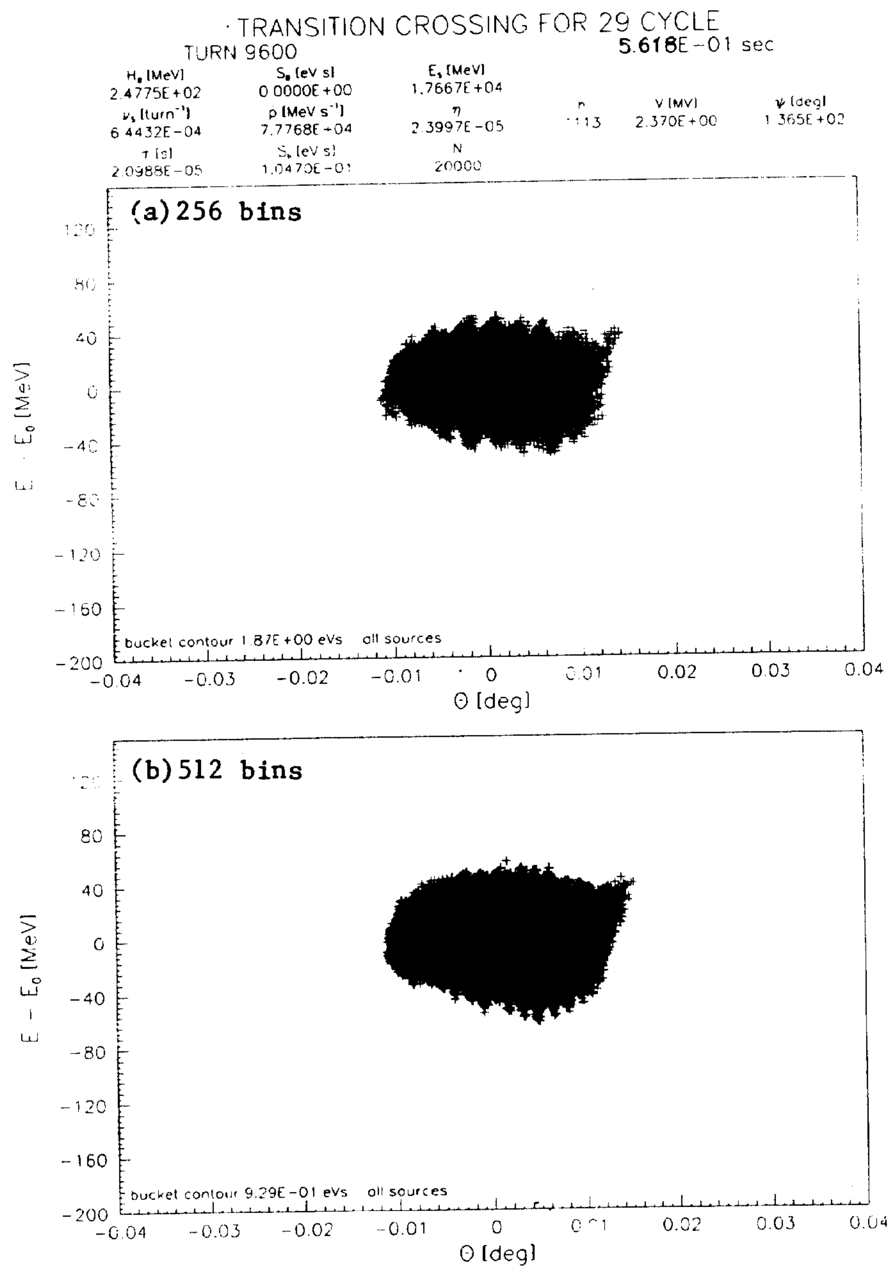

Figure 1: ESME simulations of a Fermilab Main Ring bunch containing 4×10^{10} particles with initial emittance of 0.1 eV-sec just after transition with (a) 256 bins and (b) 512 bins in an rf wavelengths; 20,000 and 160,000 macro-particles have been used in the two cases. Excitations of 6.81 and 13.6 GHz corresponding to the respective bin widths are clearly seen in the two plots.

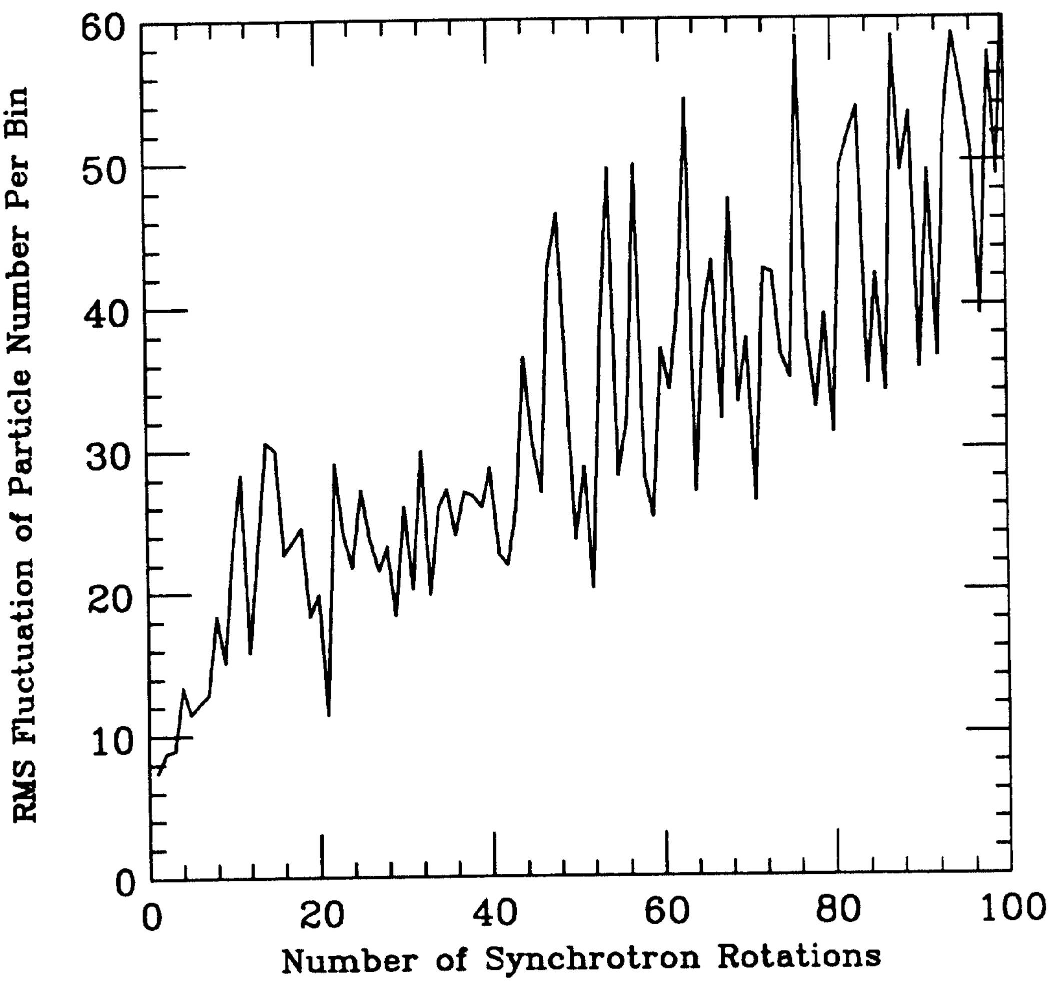

Figure 2: Plot of rms fluctuation of excess particles per bunch versus number of synchrotron rotations, showing the rapid loss of Hammersley statistics and restoration to randomness.